$$= \sqrt{2} - (n+1) \int_{-1}^{0} \frac{(x+1)^{n+2} + (x+1)^{n}}{\sqrt{x^2+2x+2}}\, dx$$

$$= \sqrt{2} - (n+1) \int_{-1}^{0} \frac{(x+1)^{n+2}}{\sqrt{x^2+2x+2}}\, dx$$

$$- (n+1) \int_{-1}^{0} \frac{(x+1)^{n}}{\sqrt{x^2+2x+2}}\, dx$$

$$U_{n+2} = \sqrt{2} - (n+1) \cdot U_{n+2} - (n+1) \cdot U_n$$

$$\Rightarrow U_{n+2} + (n+1) U_{n+2} = \sqrt{2} - (n+1) U_n$$

$$\Rightarrow U_{n+2} \cdot (n+2) = \sqrt{2} - (n+1) U_n$$

$$\Rightarrow \boxed{U_{n+2} = \frac{\sqrt{2}}{n+2} - \frac{n+1}{n+2} U_n}$$

b En déduire $\lim\limits_{n \to +\infty} u_n$.

on a (U_n) est une suite convergente soit l sa limite

on a $\lim\limits_{n \to +\infty} U_{n+2} = \lim\limits_{n \to +\infty} U_n = l$